Improving Convergence in Lattice Gauge Theory

Tanmoy Bhattacharya and Rajan Gupta (T-8), Weonjong Lee (Seoul National University, South Korea), and Stephen Sharpe (University of Washington, Seattle); ***tanmoy@lanl.gov***

The fundamental properties of particles and interactions at the scale of elementary particles is described by Quantum Field Theories, which incorporate quantum fluctuations at every point in our universe. These theories are not amenable to analytic calculations when the couplings between the various particles is strong. Numerical simulations with finite computers require the theory be discretized on lattice of finite number of points, and treat the ignored fluctuations, of length scale smaller than lattice spacing between the points, separately. Fortunately, the most important of these theories, called non-abelian gauge theories have the property of "asymptotic freedom" which allows us to calculate the effects of short distance fluctuations perturbatively. Thus, in principle, we need to only make the lattice spacing small enough to match to the scale where perturbative calculations become reliable to recover the results of the continuum theory with confidence.

We illustrate the important contributions of this approach to matching scale using the properties of the low-lying hadrons using lattice Quantum Chromodynamics (QCD). In QCD, the effective perturbation parameter α_s is about 0.2, at a lattice spacing of 0.1 fm. Even then, the nonperturbative effects are quite large, as is evident when we extract the same quantity from calculations at different lattice spacings (see Fig. 1).

There is, however, a systematic method of improving the discretization so that the errors are reduced. The leading correction in this Symanzik improvement scheme removes all errors which are proportional to the lattice spacing, leaving behind those that go as its square or higher power. Improvement involves addition of extra "improvement

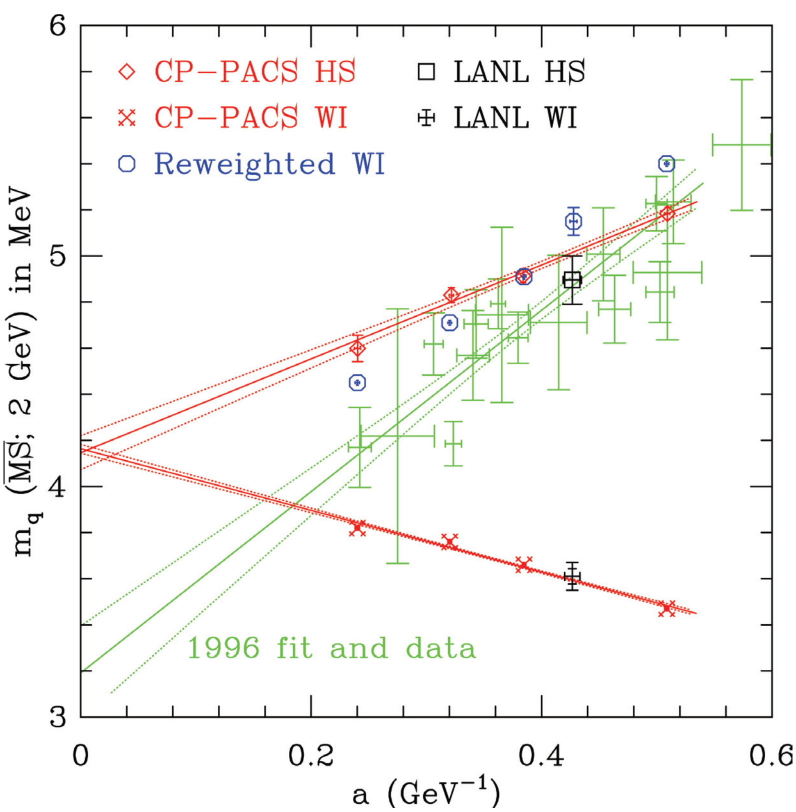

Figure 1— The average light quark mass extracted from calculations at different lattice spacings show large lattice spacing artifacts. The green line indicates the extrapolation performed in 1996 before the accurately determined red points were available. The different symbols represent different methods that are expected to agree only when the lattice spacing, a, is zero.

	$\beta = 6.2$			$\beta = 6.4$		
	LANL	**ALPHA**	**P. Th.**	**LANL**	**ALPHA**	**P. Th.**
c_{SW}	1.614	1.614	1.481	1.526	1.526	1.449
Z^o_V	0.7874(4)	+0.7922(4)(9)	+0.821	+0.802(1)	+0.8032(6)(12)	+0.830
Z^o_A	+0.818(2)(5)	+0.807(8)(2)	+0.839	+0.827(1)(4)	+0.827(8)(1)	+0.847
Z^o_P/Z^o_S	+0.884(3)(1)	N.A.	+0.959	+0.901(2)(5)	N.A.	+0.962
c_A	-0.032(3)(6)	-0.038(4)	-0.012	-0.029(2)(4)	-0.025(2)	-0.011
c_V	-0.09(2)(1)	-0.21(7)	-0.026	-0.08(1)(2)	-0.13(5)	-0.024
c_T	+0.051(7)(17)	N.A.	+0.019	+0.041(3)(23)	N.A.	+0.018
$\tilde{b}_V$	+1.30(1)(1)	N.A.	+1.099	+1.24(1)(1)	N.A.	+1.093
b_V	+1.42(1)	+1.41(2)	+1.255	+1.39(1)	+1.36(3)	+1.239
$\tilde{b}_A - \tilde{b}_V$	-0.11(3)(4)	N.A.	-0.002	-0.09(1)(1)	N.A.	-0.002
$b_A - b_V$	-0.11(3)(4)	N.A.	-0.002	-0.08(1)(1)	N.A.	-0.002
$\tilde{b}_P - \tilde{b}_S$	-0.09(2)(1)	N.A.	-0.062	-0.090(10)(1)	N.A.	-0.059
$\tilde{b}_P - \tilde{b}_A$	-0.09(3)(3)	N.A.	+0.001	-0.12(2)(5)	N.A.	+0.001
$\tilde{b}_A$	+1.19(3)(5)	N.A.	+1.097	+1.16(2)(3)	N.A.	+1.092
b_A	+1.32(3)(4)	N.A.	+1.252	+1.31(2)(1)	N.A.	+1.237
$\tilde{b}_P$	+1.23(11)(7)	N.A.	+1.099	+1.13(4)(7)	N.A.	+1.093
$\tilde{b}_S$	+1.31(10)(6)	N.A.	+1.161	+1.22(4)(8)	N.A.	+1.151

Table 1— ***Comparing the improvement coefficients obtained by LANL collaboration with previous calculations by the ALPHA collaboration and estimates from Perturbation Theory. Results are shown for two values of the parameter β of the Wilson action that sets the lattice spacing.***

operators" to both the action and the operators whose matrix elements need to be calculated. In general, the coefficients of these improvement operators have large uncertainty when perturbation theory cannot be trusted.

It was noted in QCD, however, that the leading improvement operators all break a symmetry of the theory that is recovered in the continuum limit. As a result, the non-perturbative tuning of these coefficients can be done by demanding that this "chiral symmetry" is restored to the expected order. In fact, we showed that for the lowest dimension fermion bilinear operators, all the improvement coefficients can be obtained using lattice simulations. and calculated. Our results, for all but one of these coefficients in the quenched approximation (see Table 1), are the state of the art and are being used by other collaborators in their work.